

## Chapter 5 - NEUTRON FLUX ON SAMPLE

Flux on sample is an important factor in characterizing the performance of a neutron scattering instrument. It depends on many factors as discussed here.

### 1. THE COLD NEUTRON SOURCE SPECTRUM

The liquid hydrogen neutron cold source is characterized by the following angular spectrum distribution (neutrons/cm<sup>2</sup>.s.Å.ster):

$$\frac{\varphi(\lambda)}{4\pi L_0^2} = \frac{\Phi_0}{4\pi L_0^2} \frac{2\lambda_T^4}{\lambda^5} \exp\left(-\frac{\lambda_T^2}{\lambda^2}\right) \quad (1)$$

It is also referred to as the “Maxwellian” distribution.  $\lambda$  is the neutron wavelength and  $\lambda_T$  is a cold source constant defined as  $\lambda_T = h / \sqrt{2mk_B T}$ .  $\lambda_T$  can be expressed as:

$$\lambda_T = A / \sqrt{T_e} . \quad (2)$$

The constant  $A = 30.9 \text{Å}\sqrt{\text{K}}$ ,  $T_e$  is the cold source effective temperature  $T_e = 32 \text{ K}$ . Note that the cold source real temperature is the condensation temperature of hydrogen (around 20 K). Therefore  $\lambda_T = 5.5 \text{ Å}$  is a good estimate in our case. The cold neutron wavelength distribution is therefore peaked around 3.5 Å and falls off with a  $1/\lambda^5$  tail. The normalization factor  $\Phi_0$  is determined through flux measurements.

### 2. NEUTRON FLUX ON SAMPLE

The neutron current on sample (neutrons/s) can be estimated for a typical SANS instrument configuration as:

$$\varphi(\lambda)\Delta\lambda \frac{\Delta\Omega_1}{4\pi} \frac{\Delta\Omega_2}{4\pi} = \frac{\varphi(\lambda)}{16\pi^2} \Delta\lambda \frac{A_1}{L_0^2} \frac{A_2}{L_1^2} \quad (3)$$

$\Delta\lambda$  is the wavelength spread,  $\Delta\Omega_1$  is the solid angle subtending the source aperture defined by the area  $A_1$  and  $\Delta\Omega_2$  is the solid angle subtending the sample aperture defined by the area  $A_2$ ,  $L_0$  and  $L_1$  are the cold source-to-source aperture and source aperture-to-sample aperture distances respectively.

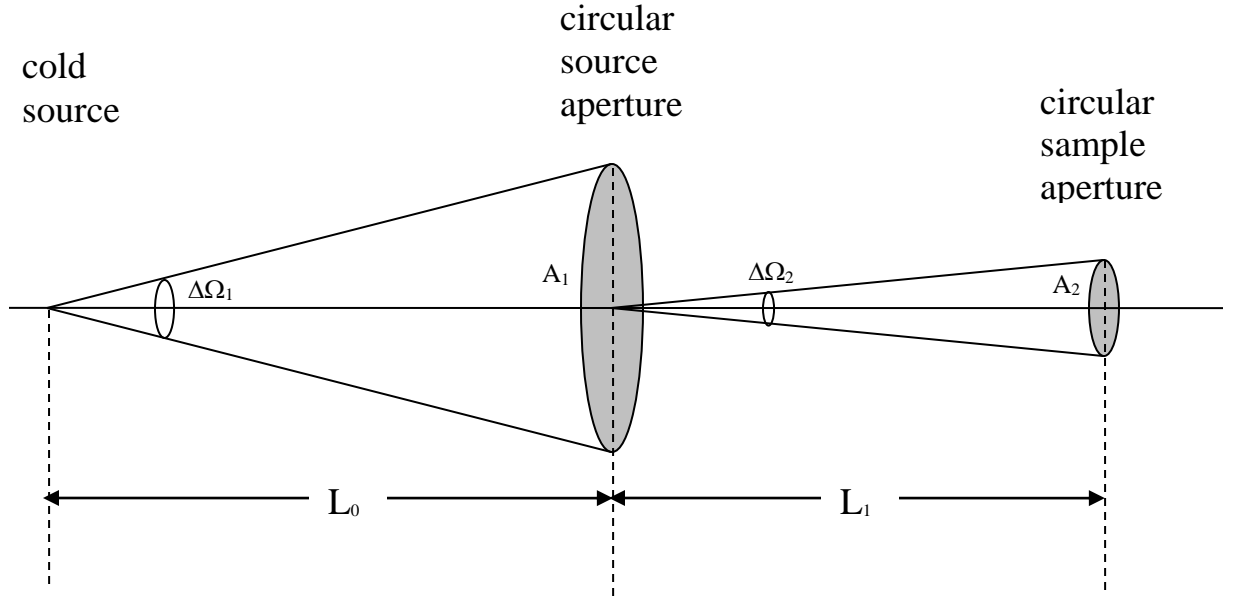


Figure 1: Typical pre-sample SANS collimation geometry. This figure is not to scale. Vertical scale is of order of centimeters while horizontal scale is of order of meters.

This quantity can be expressed as:

$$\phi(\lambda)\Delta\lambda \frac{\Delta\Omega_1}{4\pi} \frac{\Delta\Omega_2}{4\pi} = \frac{\Phi_0}{8\pi^2} \frac{\lambda_T^4}{\lambda^5} \exp\left(-\frac{\lambda_T^2}{\lambda^2}\right) \Delta\lambda \frac{A_1}{L_0^2} \frac{A_2}{L_1^2} \quad (4)$$

with  $\lambda_T = 5.5 \text{ \AA}$ . In order to make the neutron flux expression match the measured flux at the NG3 SANS instrument the following factor is chosen:

$$\frac{\Phi_0}{8\pi^2 L_0^2} = 1.65 * 10^{12} \text{ n/cm}^2 \cdot \text{s} . \quad (5)$$

The estimated **flux** (or **current density**) **on sample** ( $\text{n/cm}^2 \cdot \text{s}$ ) is given by:

$$\phi(\lambda) = \frac{\phi(\lambda)}{A_2} \Delta\lambda \frac{\Delta\Omega_1}{4\pi} \frac{\Delta\Omega_2}{4\pi} = \frac{\Phi_0}{8\pi^2 L_0^2} \frac{\lambda_T^4}{\lambda^4} \exp\left(-\frac{\lambda_T^2}{\lambda^2}\right) \left(\frac{\Delta\lambda}{\lambda}\right) \left(\frac{A_1}{L_1^2}\right) \quad (6)$$

$$\phi(\lambda) = \frac{1.507 * 10^{15}}{\lambda^4} \exp\left(-\frac{30.25}{\lambda^2}\right) \left(\frac{\Delta\lambda}{\lambda}\right) \left(\frac{A_1}{L_1^2}\right)$$

Consider a typical neutron wavelength and wavelength spread:

Neutron wavelength:  $\lambda = 6 \text{ \AA}$ .  
Wavelength spread:  $\Delta\lambda/\lambda = 0.15$ .

So that:

$$\phi(6 \text{ \AA}) = 7.53 * 10^{10} \left( \frac{A_1}{L_1^2} \right) \text{ n/cm}^2.\text{s} \quad (7)$$

This expression is used in the following section.

### 3. CASE OF SPECIFIC CONFIGURATIONS

Consider two instrument configurations both using:

Neutron wavelength:  $\lambda = 6 \text{ \AA}$ .  
Wavelength spread:  $\Delta\lambda/\lambda = 0.15$ .

The first configuration corresponds to high flux on sample:

Source aperture radius:  $R_1 = 2.5 \text{ cm}$ .  
Area of source aperture:  $A_1 = \pi 2.5^2 = 19.63 \text{ cm}^2$ .  
Source-to-sample distance:  $L_1 = 3.82 \text{ m}$ .

So that  $\phi(6\text{\AA}) = 1.01 * 10^7 \text{ n/cm}^2.\text{s}$  for the high flux configuration.

The second configuration corresponds to low flux on sample:

Source aperture radius:  $R_1 = 1.9 \text{ cm}$ .  
Area of source aperture:  $A_1 = \pi 1.9^2 = 11.34 \text{ cm}^2$ .  
Source-to-sample distance:  $L_1 = 16.22 \text{ m}$ .

So that  $\phi(6\text{\AA}) = 3.24 * 10^5 \text{ n/cm}^2.\text{s}$  for the low flux configuration.

### 4. MEASURED FLUX ON SAMPLE

The two previously considered cases correspond to two specific configurations on the NG3 30 m-SANS instrument at NIST. Flux on sample measurements were made for these two configurations described above and for a range of wavelengths. These results are plotted here.

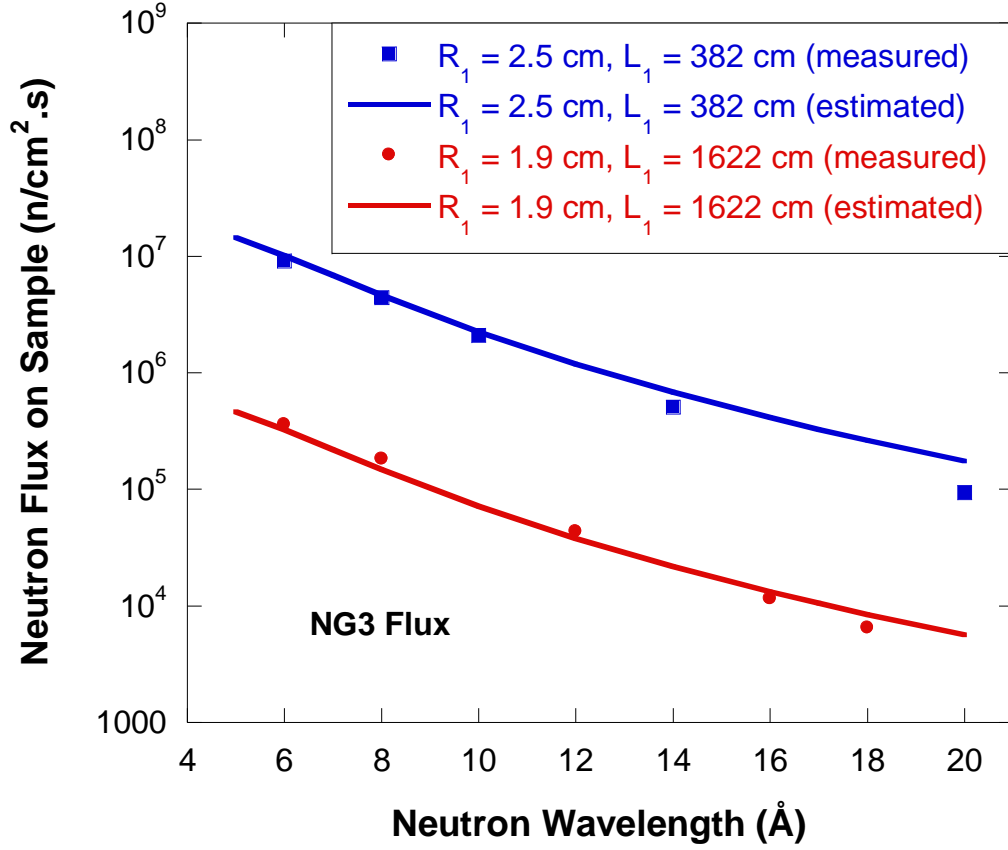


Figure 2: Measured neutron flux on sample with varying wavelength for the high flux configuration ( $R_1 = 2.5$  cm,  $L_1 = 3.82$  m) and the low flux configuration ( $R_1 = 1.9$  cm,  $L_1 = 16.22$  m). Estimates values are also plotted.

Note that the neutron current on sample (n/s) is obtained by multiplying the neutron flux by the area of the sample aperture  $A_2 (= \pi R_2^2)$ . In our notation, that quantity is given by  $\Phi(\lambda) = \phi(\lambda)A_2$ . Note that  $\Phi(\lambda)$  and  $\phi(\lambda)$  are not per unit wavelength, but are calculated at wavelength  $\lambda$ .

Considering a sample aperture of radius  $R_2 = 0.635$  cm, the following neutron currents can be estimated:

$$\Phi(6\text{\AA}) = 1.28 \cdot 10^7 \text{ n/s for the high flux configuration.}$$

$$\Phi(6\text{\AA}) = 4.10 \cdot 10^5 \text{ n/s for the low flux configuration.}$$

These are reasonably high numbers for a SANS instrument (Cook et al, 2005).

## 5. NEUTRON BEAM MONITOR COUNT RATE

The neutron beam monitor count rate is measured on a regular basis for increasing wavelength. Measurements shown here were taken on the NG3 30 m SANS instrument at the NIST CNR before the optical filter was installed. The beam monitor is a low-efficiency fission counter and is placed just after the velocity selector. It detects neutrons through their absorption in a thin U-235 plate. The absorption cross section varies like “1/v” (v being the neutron velocity). It is proportional to the neutron wavelength  $\lambda$ , i.e.,  $\sigma_a(\lambda) = c\lambda$  where c is a constant.

The measured monitor count rate  $m(\lambda)$  is compared to the following empirical expression:

$$m(\lambda) = \frac{2.25 * 10^7}{\lambda^3} \exp \left[ - \left( \frac{7.37}{\lambda} \right)^2 \right]. \quad (8)$$

The multiplicative constant depends on the fission counter used. Note the characteristic  $\lambda$ -dependence. The tail drops out like  $1/\lambda^3$ . Recall that the cold source spectrum drops out like  $1/\lambda^5$ . Use of a velocity selector (with constant  $\Delta\lambda/\lambda$ ) changes the tail of the transmitted spectrum to  $1/\lambda^4$ . Therefore, the tail of the corrected monitor count rate varies like  $m(\lambda)/\sigma_a(\lambda) \sim 1/\lambda^4$  where  $\sigma_a(\lambda)$  is the neutron absorption cross section. The wavelength dependence of the monitor count rate/wavelength and the neutron current density are the same. It is not clear as to why the constants in the exponential are different.

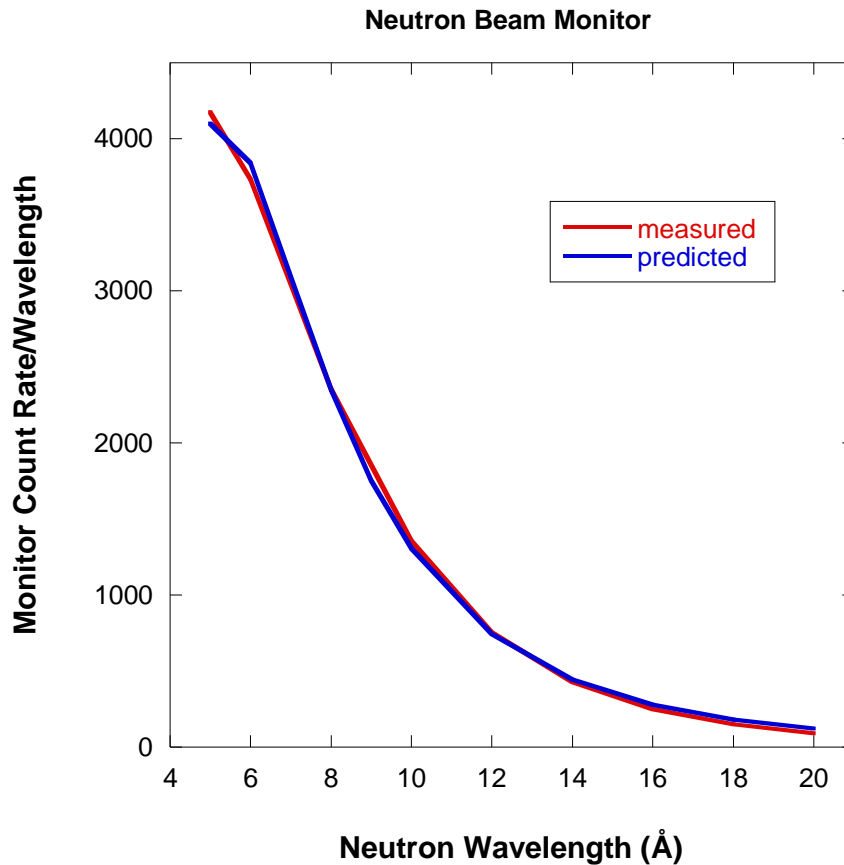


Figure 3: Variation of the neutron beam monitor count rate divided by the neutron wavelength with increasing wavelength.

## REFERENCE

J.C. Cook, C.J. Glinka, and I.G. Schroder, "Performance of the vertical optical filter for the NG-3 30 m SANS instrument at the National Institute of Standards and Technology's Center for Neutron Research", *Review of Scientific Instruments*, 76, no. 2, 25108-1-8, (2005).

## QUESTIONS

1. What is the neutron current?
2. What is the neutron flux (or current density) at the sample?
3. What is the highest neutron flux on sample for 6 Å neutrons at the NG3 SANS instrument?
4. How do neutron fluxes compare with x-ray fluxes?
5. Is the neutron current crossing the sample aperture the same as the detector count rate?

## ANSWERS

1. The neutron current is the number of neutrons per second.
2. The neutron flux at the sample is expressed in  $\text{n/cm}^2\cdot\text{s}$ . It is independent of sample area.
3. The highest neutron flux on sample for 6 Å neutrons at the NG3 SANS instrument is around  $10^7 \text{ n/cm}^2\cdot\text{s}$ . It is obtained for a high-Q high flux configuration.
4. Neutron fluxes are orders of magnitude lower than x-ray fluxes. Even fluxes for a rotating anode x-ray source are higher than the highest neutron source fluxes.
5. The neutron current crossing the sample aperture is not the same as the detector count rate because of loss due to attenuation in the scattering flight path, due to neutrons that are scattered outside of the detector solid angle and due to the detector absorption cross section and non-perfect detector efficiency.